

BAYESIAN REGRESSION OVER SPARSE FATIGUE CRACK GROWTH DATA FOR NUCLEAR PIPING

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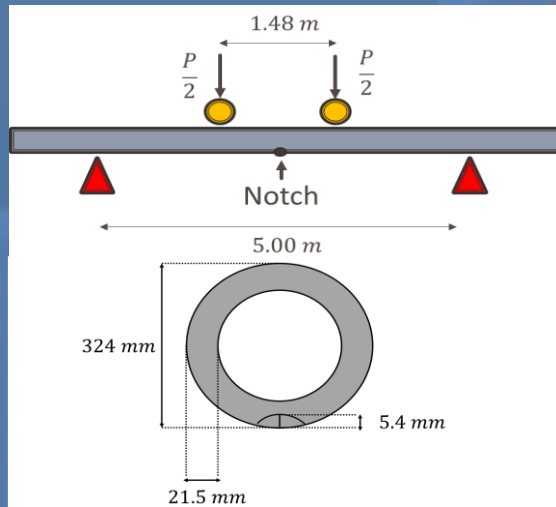
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INTRODUCTION

- Research objectives: To perform probabilistic model calibration and quantify the uncertainty over the sparse data;
- A set of crack data obtained via a 4-point bending test on a Carbon-Steel nuclear piping [1];
- Test was conducted over 40000 periodic stress cycles, N_{cycles} ;
- Each stress cycle has stress range: $\Delta P = 156 \text{ MPa}$;
- 24 readings of crack depth, z , obtained for 24 distinct N_{cycles} .



PROBLEM

- Crack growth assumed to follow Paris-Erdogan Law [2]:

$$\frac{dz}{dN_{cycles}} = C (\Delta K)^m \quad (1)$$

- This can be linearized to:

$$\log \left[\frac{dz}{dN_{cycles}} \right] = m \cdot \log[\Delta K] + \log[C] \quad (2)$$

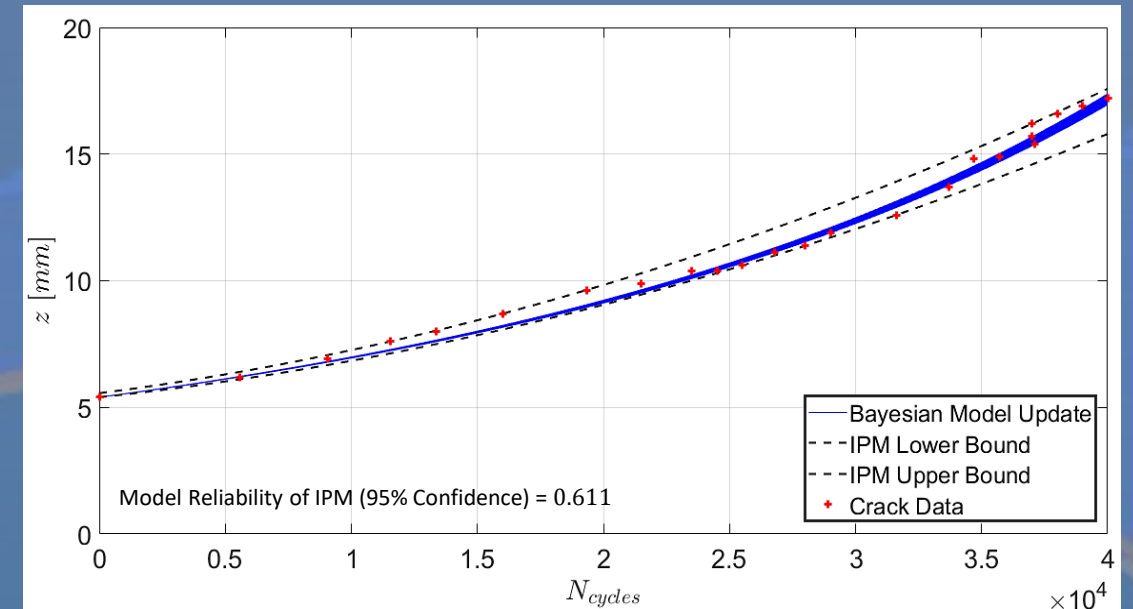
where:

$$\Delta K = \left(\frac{z}{\sqrt{r_m t}} \right)^{\frac{1}{8}} \cdot \Delta P \cdot \sqrt{\pi \cdot z} \quad (3)$$

METHODOLOGY

- Bayesian regression technique for uncertainty quantification over the sparse data in log-space;
- Epistemic parameters to be inferred: $\theta = \{\log[C], m\}$;
 - Prior PDF: 2D Uniform distribution with correlation coefficient of -0.999 defined by a Gaussian Copula. $\log[C]$ has bounds $[-50, 0]$ while m has bounds $[0, 10]$;
 - Likelihood function is Gaussian with standard deviation: $\sigma = 0.0191$;
 - Model used for Bayesian updating is defined by Eq. (2) ;
 - 1000 samples generated via TMCMC [3].
- Compare the results with 2nd-order polynomial Interval Predictor Model [4] in the original real space.

RESULTS



CONCLUSION

- Bayesian regression / model updating results showed that all possible trajectories lie within IPM;
- Bayesian model updating results yield tighter bounds and this is attributed to the choice of σ in the Likelihood function;
- Further works: To compare the results from Bayesian regression using different models of ΔK and to also include Kriging as a form of validation.

REFERENCES

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